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"Despite global fundraising slowing, $2.1 trillion in Dry Powder well positions US VC and PE to take on Global opportunities, but at what return?"

F. John Mathis, Editor

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Alexander Rudin, Jason Mao, Nan R. Zhang, and Anne-Marie Fink

PORTFOLIO MANAGEMENT RESEARCH

Private Equity Program Breadth and Strategic Asset Allocation

ALEXANDER RUDIN, JASON MAO, NAN R. ZHANG, AND ANNE-MARIE FINK

ALEXANDER RUDIN
is a managing director at State Street Global Advisors in Boston, MA.
alexander_rudin@ssga.com

JASON MAO
is a vice president at State Street Global Exchange in Boston, MA.
xmao@statestreet.com

NAN R. ZHANG
is a vice president at State Street Global Exchange in Cambridge MA.
nzhang2@statestreet.com

ANNE-MARIE FINK
is a managing director at State Street Global Advisors in Stamford, CT.
anne-marie_fink@ssga.com

While institutional investors often see clear benefits in allocating part of their portfolio to private equity, developing a rational asset allocation framework in this space is challenging. Most traditional asset allocation methods are rooted in modern portfolio theory, which is based on two implicit assumptions: (1) asset return and risk forecasts are known and (2) the portfolio can be rebalanced at any time. Both assumptions are invalid in the case of private equity.

Investors allocate to newly formed blind pools, with managers' track records based on prior funds that began years or even decades ago. For private equity overall, index return streams are based on broad-based swaths of the private equity industry, while the idiosyncratic risks of narrower baskets of blind pools are largely side-stepped.

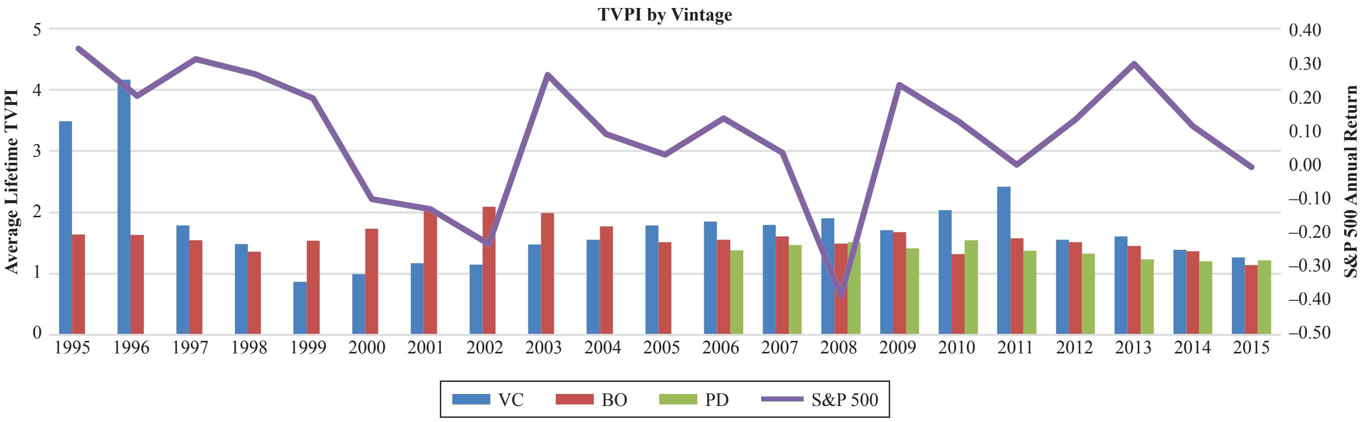
The liquidity of private equity funds is also extremely poor, making rebalancing a challenge. While a secondary market for private equity funds exists, it is shallow and trading is very expensive. Whenever an institutional investor commits to a private equity fund, this decision is final in the sense that the investor is likely and highly incented to hold such a fund until its complete liquidation 10 or more years later, regardless of the fund's performance.

These circumstances have prompted many institutional investors that invest in or manage a private equity program to turn away from traditional, Sharpe ratio oriented asset allocation frameworks, and to focus instead on achieving attractive total returns over the life of private equity funds. One of the most widely used performance measures for a private equity portfolio over the life of the program is the TVPI (Total Value to Paid In) multiple. It measures the total value created by a fund and can be calculated by dividing the sum of cumulative fund distributions and residual fund value by paid-in capital. The advantage of using TVPI is that it encompasses all valuation information typically available for a fund: the cash flows and the valuation of what remains within the pool.

In all, TVPI has become one of the most widely used measures of private equity program success. Consultants gauge a private equity program based on its relative performance to the strategy peer group. Some private equity programs target a certain level of TVPI as an explicit investment objective. Since there is an expectation that the program manager will deliver a competitive TVPI over the program life, it is attractive to build a framework for the strategic asset allocation in the private equity space that is both anchored in this measure and consistent with the empirical facts about it. However, lack of reliable data has historically stymied research efforts to study empirical properties of private equity funds from a TVPI perspective.

Exhibit 1

Private Equity TVPI by Vintage vs. S&P 500 Returns



Derived from actual cash flow data of State Street's limited partner clients who make commitments to private equity funds, SSPE is one of the most detailed and accurate private equity data sets in the industry today. Because this dataset does not depend on voluntary reporting of information, it is less exposed to biases common among other industry indexes, resulting in more reliable and consistent content. As of Q4 2017, SSPE comprised more than 2,800 funds representing more than $2.7 trillion in capital commitments, with cash flow data back to 1980. Studying such a dataset on an anonymized basis provides an opportunity to gain analytical insight into an otherwise opaque asset class.

HISTORICAL PRIVATE EQUITY RETURNS THROUGH THE TVPI LENS

Our first goal is to study the properties of lifetime TVPIs for individual private equity funds by vintage. The results show that they do vary somewhat, but less than one would expect as shown in Exhibit 1. After the mid-1990s, when venture capital funds demonstrated outstanding success, TVPI levels stabilized and—on average—have shown remarkable and somewhat unexpected[1] resilience to public equity market gyrations.

While Exhibit 1 suggests that diversification of a private equity program across vintages is important, it does not provide any information on how dispersed the fund results are within the vintages. Individual private equity funds tend to have 10 to 50 deals within the portfolio, much fewer than in most of the traditional public equity funds. Also, managers of private equity funds do not usually attempt to minimize tracking error of their portfolio for a given benchmark, public or private. All in all, fund risks may be expected to have a high idiosyncratic component that can be diversified away.

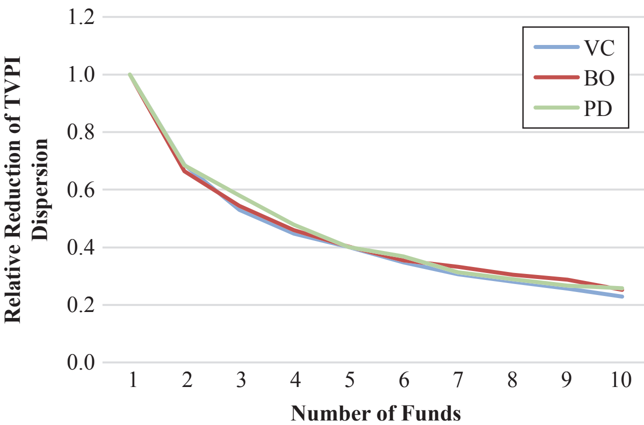
To study this phenomenon, we randomly selected baskets of funds within each vintage year and strategy and in the course of a simulation analyzed how the dispersion of a same-size basket of funds scales down (relative to a single-fund dispersion value), as the number of funds in this basket increases. Exhibit 2 shows results of this study; risk reduction is nothing short of dramatic, which explains why large private equity programs tend to have hundreds of funds in their portfolios.[2]

[1] Part of this resilience comes out of the simple fact that a typical fund's lifetime is 10+ years. Funds in adjacent vintages are operating in highly overlapping environments, so it is intuitive to expect TVPIs of such funds to be linked to each other. Indeed, serial correlation analysis shows strong serial correlation with the time series of average TVPIs by vintage. That said, we were surprised by our inability to detect linkages between TVPI results for a given vintage and stock market performance that year (or the year after). Perhaps, this lack of correlation is related to fund managers implicitly (or explicitly) targeting TVPI as their investment goal and managing timing of the exit to compensate for any gyrations in their fund TVPI caused by the stock market volatility within the vintage year.

[2] This circumstance may temper conclusions made in the recent work by McKay, Shapiro, and Thomas (2018), where the largest pension funds allocating to hundreds of external funds were deemed "over-diversified." Those conclusions were based on diversification studies across US large-cap blend, large-cap value, and

Exhibit 2

Diversification Benefits across Strategies



TVPI-TARGETING AND STRATEGIC ASSET ALLOCATION

Given the 10+ year lifecycle of private equity vehicles and effective lack of secondary market for them, private equity investing is a long-term game. Institutions that plan to enter the space must inevitably make certain strategic choices, including deciding on the private equity strategy mix and also the breadth of the portfolio they will aim to pursue. Both choices are not easy to make. Investing into venture capital and buyout strategies requires somewhat different skills sets. Further, increasing the breadth of the portfolio by expanding the roster of funds is expensive, as additional fund investments come with incremental costs.

One of the goals of our research was to establish a framework that helps make those strategic choices in a way that would be consistent with empirical facts about private equity fund returns and risks. With that in mind, we are now shifting beyond a vintage-by-vintage TVPI dynamic into comparing "mini-programs" in private equity that encompass long time periods and that basically imitate the behavior of typical private equity investors. Since private debt is a newer strategy with less data available for analysis, we will focus exclusively on venture capital and buyout strategies.

We created private equity "mini-programs" in the following way:

(1) Each mini-program covered 20 years between 1995 and 2014.[3]
(2) All programs assumed a $1 annual commitment allocated evenly across N randomly selected funds from the corresponding vintage year (selected without repetition).

Five hundred such programs were simulated for each strategy to study the statistical behavior of a private equity strategy as a function of the number of funds. Exhibit 3 illustrates the TVPI distributions for the most basic case of allocating to a single fund per each vintage year.

The differences between the two distributions are stark. Venture capital programs carry a promise of somewhat higher returns, at least in the TVPI sense, but these returns come with significantly higher dispersion. Interestingly, not only are the widths of the two distributions substantially different, but their shapes are as well. Most of venture outperformance comes from a few "lottery-like" outcomes, leading to a distribution that is significantly skewed to the right. The buyout distribution is much more symmetrical.

As was the case for within vintage TVPI results, both distributions narrow as the breadth of our mini-programs increases. Exhibit 4 illustrates the extent of standard deviation compression; skewness compresses in a similar fashion.

With the statistical properties of our mini-programs established, we can begin to build the framework for making strategic asset allocation decisions, starting with choosing the investment objective. From the very beginning, our analysis centers on TVPI, so we formulate an investment objective that aims to *maximize our chances of meeting or exceeding a target level of TVPI:*

$$\text{Objective: } max(Probability(TVPI_{Program} \geq TVPI_{Target})) \quad (1)$$

Our goal is to estimate and maximize the probability of reaching this objective on the overall program level.

large-cap growth public equity funds. Active share of baskets of such funds indeed drop very quickly as baskets increase in size. Private equity funds from the very beginning have a higher idiosyncratic component requiring broader diversified portfolios. Additionally, the long, 10+ year lifecycles of those funds combined with the need for vintage diversification guarantees that even a moderately diversified portfolio (say, 3–4 funds per vintage year) translates into 50+ external fund investments in private equity funds alone, perhaps partly explaining why large pension fund portfolios contain so many private equity items within them.

[3] The choice of the beginning year was driven by the availability of data. 2014 was chosen as the terminal year because we wanted at least three full years of seasoning in the funds.

Exhibit 3

Differences between Venture Capital and Buyout Funds

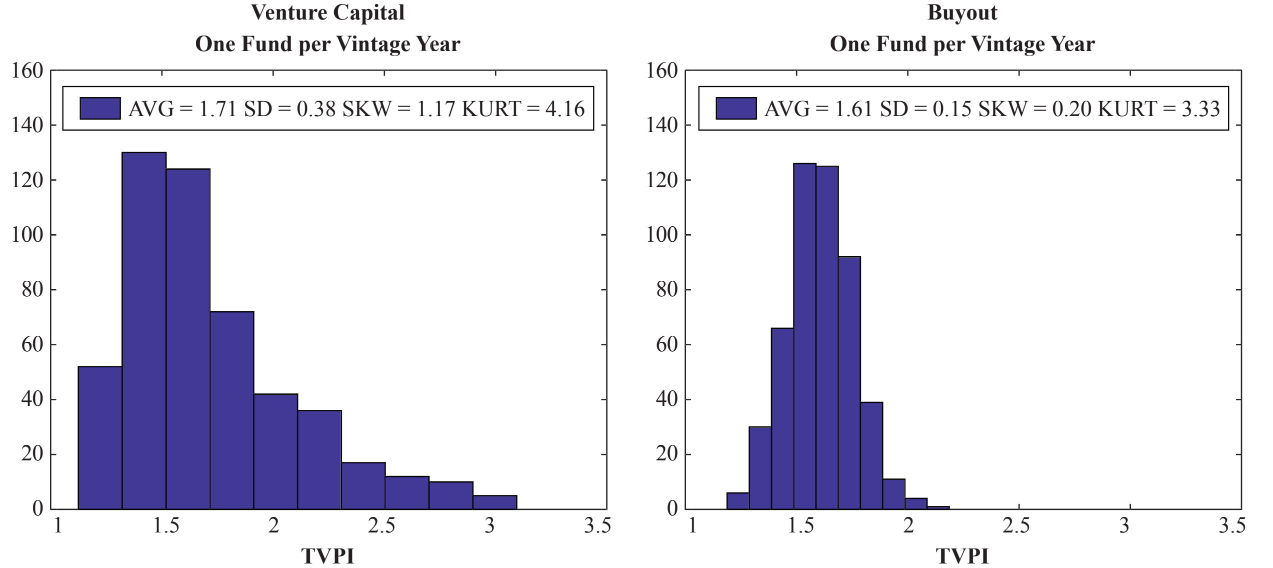
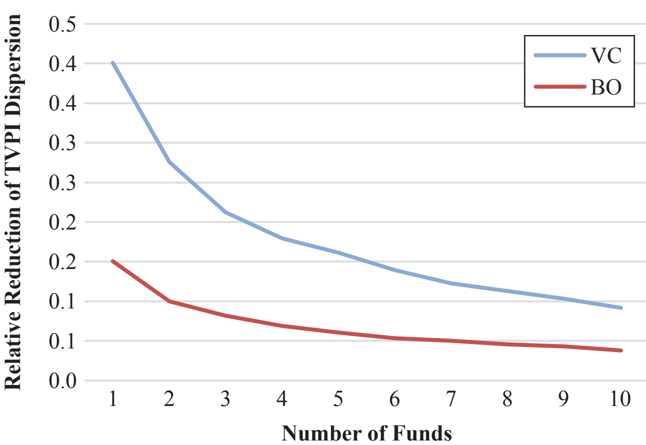


Exhibit 4

Risk Reduction and Mini-Program Breadth



To do this, we need to build a distribution of the program outcomes and compare it with our target. We depict the process schematically in Exhibit 5.

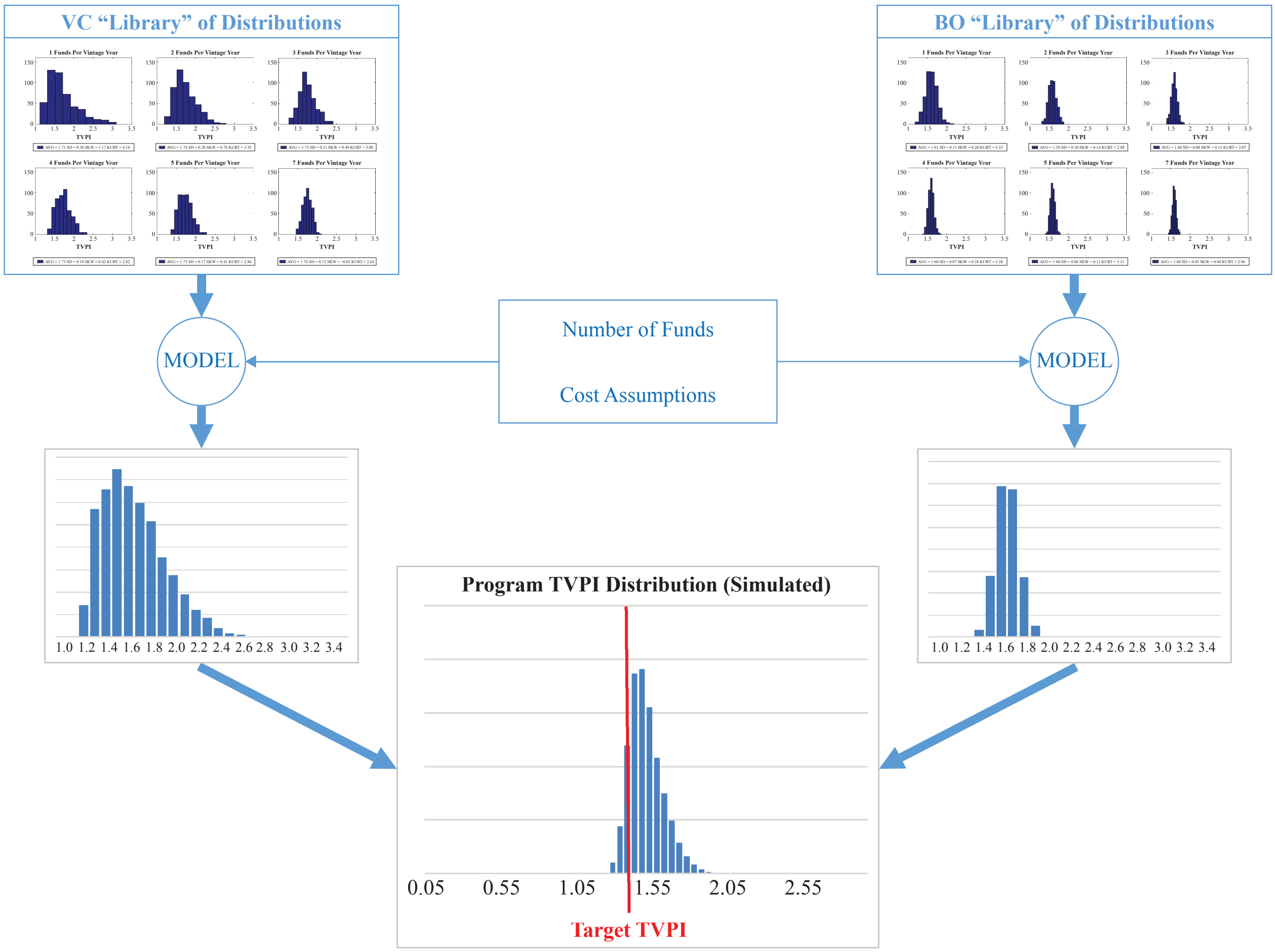
The first challenge with approaching this task analytically is that the distributions of our mini-program components are complex and unknown. We overcome this challenge by using a method that allows us to generate multivariate data in a way that fits observed *moments* of the empirical distribution, while bypassing the question of the precise shape of that distribution. The Appendix contains details of the methodology, which was pioneered by Fleishman (1978) and has since been used in various econometric studies (see, for example, Lyhagen 2001).

The second challenge is the need to incorporate incremental costs associated with increasing the breadth of such mini-programs. We do this in the most straightforward way possible by assigning a penalty to the expected TVPI value for a mini-program that is proportional to the number of funds in such a mini-program.

We then approach the problem numerically using our historically observed risk characteristics (variance, skewness, and kurtosis) of venture and buyout mini-programs to calibrate the simulation. Results of this analysis for a few illustrative parameter combinations are shown in Exhibit 6.

When costs are disregarded and VC and BO mini-programs are assumed to have the same expected value of TVPI, a diversified buyout strategy is the clear winner (Exhibit 6A). Incorporating non-zero costs shows off the trade-off between costs and the desire to diversify, with a *narrowly* diversified, primarily buyout program emerging as optimal (Exhibit 6B). If venture capital mini-programs are assumed to have a return advantage over buyout strategies—while retaining its disadvantage in terms of risk—a mixed strategy emerges as a winner (Exhibit 6C).

EXHIBIT 5
Distribution of Program Outcomes and Target TVPI



The final example (Exhibit 6D) aims to capture probabilities of portfolio outcomes when our chosen "minimal" TVPI is quite high, significantly exceeding both VC and BO average expectations. High uncertainty of VC outcomes combined with pronounced positive skewness of their distribution makes concentrated VC portfolios deliver a non-zero (albeit still low) probability of reaching such outcomes. Investors attracted to such "lottery-like" outcomes may turn to venture capital strategies to a larger degree than those who are seeking high-confidence situations and are willing to accept lower returns.[4]

TAKEAWAYS AND CONCLUDING REMARKS

Historical TVPIs are a standard tool, used by consultants and program managers alike, to assess private equity program results. Typically, such analysis involves calculating the TVPI of a program by vintage, and then comparing this value with the fund peer group from a full private equity universe provided by Burgiss or Cambridge Associates. Results above the median and especially within the first quartile are considered attrac-

[4] One may even consider changing the investment objective from Equation 1 to a form that reflects such investor's desire. For example, one could use an objective that blends the target threshold with a "lottery-like" one and choose strategic asset allocation with that in mind:

$$max(\theta * Probability(TVPI_{Program} > TVPI_{Target}) + (1-\theta) * Probability(TVPI_{Program} > TVPI_{Lottery})).$$

EXHIBIT 6A

VC Expected TVPI = 1.6; BO Expected TVPI = 1.6; Cost = 0; Target TVPI = 1.5

Probability (TVPI > Min Target)										**Min TVPI Target**	1.5
	Number of BO Managers (per year)										
Number of VC Funds (per year)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0		0.74	0.84	0.89	0.93	0.96	0.97	0.98	0.99	1.00	1.00
1	0.52	0.62	0.72	0.80	0.85	0.89	0.93	0.95	0.97	0.98	
2	0.59	0.66	0.73	0.79	0.83	0.87	0.91	0.93	0.96		
3	0.64	0.69	0.76	0.81	0.84	0.88	0.91	0.93			
4	0.69	0.73	0.78	0.82	0.85	0.87	0.90				
5	0.73	0.76	0.80	0.82	0.86	0.88					
6	0.76	0.79	0.83	0.85	0.88						
7	0.79	0.81	0.85	0.88							
8	0.81	0.84	0.86								
9	0.83	0.86									
10	0.86										

EXHIBIT 6B

VC Expected TVPI = 1.6; BO Expected TVPI = 1.6; Cost = 0.02; Target TVPI = 1.5

Probability (TVPI > Min Target)										**Min TVPI Target**	1.5
	Number of BO Managers (per year)										
Number of VC Funds (per year)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0		0.69	0.72	0.70	0.62	0.49	0.35	0.21	0.10	0.04	0.00
1	0.50	0.58	0.62	0.63	0.59	0.53	0.44	0.33	0.22	0.13	
2	0.52	0.60	0.62	0.62	0.59	0.52	0.47	0.37	0.28		
3	0.51	0.59	0.60	0.60	0.58	0.54	0.46	0.39			
4	0.54	0.58	0.60	0.60	0.57	0.52	0.47				
5	0.48	0.52	0.54	0.54	0.52	0.49					
6	0.44	0.49	0.49	0.50	0.48						
7	0.35	0.39	0.40	0.41							
8	0.29	0.32	0.34								
9	0.21	0.24									
10	0.13										

tive. Conversely, program managers are often penalized or discarded when their results dip into the bottom quartile, even for a small fraction of vintages.

We performed an empirical analysis of private equity funds' historical success (measured in terms of TVPI), while accounting for program diversification. Our first takeaway is that comparing a "typical" program to peer group averages is somewhat unfair, as such methodology does not correct for the private equity program breadth. Single fund peer group analysis, by design, reflects performance of single private equity funds for each vintage. Private equity programs, however, usually allocate to more than one fund per vintage. That brings more diversification benefits (individual fund TVPIs within the same vintage year are generally independent) into such programs and hence lowers program TVPI *dispersion*. As a result, larger, more diversified programs generally have a much smaller chance to dip into the last quartile when compared with smaller programs. Large program managers thus seem to have a perceived edge in manager selection skills over small program managers from the analysis's perspective, arguably without merit.

It is advisable—and our empirical studies provide a basis for implementation—to create a custom "synthetic" peer group for each private equity program that mirrors the program's strategy mix and number of fund commitments *actually made each year.* This enhanced methodology corrects for the program breadth bias and creates a fairer process for assessing a program manager's skill.

EXHIBIT 6C
VC Expected TVPI = 1.7; BO Expected TVPI = 1.6; Cost = 0.02; Target TVPI = 1.5

Probability (TVPI > Min Target)										Min TVPI Target	1.5
	Number of BO Managers (per year)										
Number of VC Funds (per year)	0	1	2	3	4	5	6	7	8	9	10
0		0.69	0.71	0.70	0.62	0.48	0.36	0.21	0.09	0.04	0.00
1	0.60	0.68	0.72	0.72	0.68	0.61	0.51	0.40	0.27	0.17	
2	0.67	0.74	0.75	0.75	0.72	0.67	0.59	0.49	0.38		
3	0.71	0.76	0.77	0.76	0.74	0.70	0.63	0.55			
4	0.74	0.76	0.77	0.78	0.75	0.72	0.68				
5	0.73	0.76	0.76	0.76	0.75	0.71					
6	0.72	0.75	0.76	0.74	0.74						
7	0.67	0.70	0.71	0.72							
8	0.64	0.67	0.67								
9	0.58	0.61									
10	0.49										

EXHIBIT 6D
VC Expected TVPI = 1.6; BO Expected TVPI = 1.6; Cost = 0.02; Target TVPI = 2.0

Probability (TVPI > High Target)										High TVPI Target	2
	Number of BO Managers (per year)										
Number of VC Funds (per year)	0	1	2	3	4	5	6	7	8	9	10
0		0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.16	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2	0.07	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00		
3	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00			
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00				
5	0.00	0.00	0.00	0.00	0.00	0.00					
6	0.00	0.00	0.00	0.00	0.00						
7	0.00	0.00	0.00	0.00							
8	0.00	0.00	0.00								
9	0.00	0.00									
10	0.00										

Our second contribution entails the development of a probabilistic framework that helps make strategic trade-offs between program diversification, strategy mix, and associated (program manager-specific) costs in a rigorous, quantitative way. While somewhat involved from a technical perspective—owing to the complex statistical properties of our "assets"—this framework is intuitive and will hopefully aid institutional investors in planning their private equity programs.

APPENDIX

In this section, we outline a method for generating single variable samples of data with pre-determined first four moments of the distribution. Since we consider VC and BO mini-programs to be uncorrelated, we simulate them separately and from the sample-generating perspective we only need to consider the single variable case. However, the approach can easily be expanded to include multiple dependent variables (see Fleishman 1978; Lyhagen 2001).

Let the task be to simulate random variable Y with the first four moments (mean, variance, skewness, and kurtosis) equal to those of the empirically observed, unknown distribution. We denote those moments $m_1^Y, m_2^Y, m_3^Y, m_4^Y$. Fleishman suggested to generate data according to the following scheme

$$Y = a + bX + cX^2 + dX^3, \qquad \text{A-1}$$

where X is a standard normal variable. The distribution of Y is generally unknown, but we can (in most cases) find such a, b, c, and d, that the moments of Y will match the desired ones.

To simplify our math, we assume average $\langle Y \rangle = 0$ for the subsequent calculations. Needless to say, one can reintroduce the observed distribution's mean back into generated sample at the very end.

The p-th moment of the distribution for Y can be directly calculated using Equation A-1:

$$\begin{aligned}\langle Y^p \rangle &= m_p^Y \\ &= \left\langle (a + bX + cX^2 + dX^3)^p \right\rangle \\ &= \left\langle \sum_{\substack{k_1,k_2,k_3,k_4 \geq 0 \\ k_1+k_2+k_3+k_4=p}} C^p_{k_1,k_2,k_3,k_4} a^{k_1} (bX)^{k_2} (cX^2)^{k_3} (dX^3)^{k_4} \right\rangle\end{aligned} \quad \text{A-2}$$

with p between 1 and 4. Here we used the expression for Newton's multinomial

$$\begin{aligned}&(x_1 + x_2 + \cdots + x_m)^p \\ &\quad = \sum_{\substack{k_1 \ldots k_m \geq 0 \\ k_1+\cdots+k_m=p}} C^p_{k_1 \ldots k_m} x_1^{k_1} \ldots x_1^{k_m}, \text{ and } C^p_{k_1,k_2,k_3,k_4} \\ &\quad = \frac{p!}{k_1! \ldots k_m!}\end{aligned}$$

Using (known) values for the moments of a standard normal distribution, we can translate Equation A-2 into the system of equations for a, b, c, and d that can be solved numerically:

$$\begin{bmatrix} 0 \\ m_2^Y \\ m_3^Y \\ m_4^Y \end{bmatrix} = \begin{bmatrix} a + c \\ 2a^2 + b^2 + 6bd + 15d^2 \\ -8a^3 - 6ab^2 - 72abd - 270ad^2 \\ 60a^4 + 60a^2b^2 + 3b^4 + 936a^2bd \\ \quad + 60b^3d + 4500a^2d^2 + 630b^2d^2 \\ \quad + 3780bd^3 + 10395d^4 \end{bmatrix} \quad \text{A-3}$$

The process of generating a random sample that has moments matching the desired ones is then to (1) solve Equation A-3 to obtain appropriate values for a, b, c, and d, (2) generate a sample for the standard normal variable X, and (3) translate X sample into the desired Y sample using Equation A-1.

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